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Axioms and Intuitions about Societal Inequality What does the Gini Coefficient Measure?[1]

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Thanks to Rajiv Sethi and Debraj Ray for your illuminating contributions. Our paper (Bowles and Carlin 2020) and our contribution below present an alternative way of looking at the Gini coefficient. Before turning to the substantive issues, we provide three pieces of background – about how we came to work on this topic four years ago, about Corrado Gini and his coefficient, and about our view concerning ethics and the measurement of inequality.

Background

One of the author team in the CORE-Econ project (Curriculum Open-access Resources in Economics)[2], Antonio Cabrales, had assigned a problem set on computing inequality measures to his students in the CORE-based intro course at University College London. He had given them the conventional definition of the Gini coefficient, namely (for consistency, we use the same notation as in Bowles and Carlin 2020),

$$G^L = \frac{\sum_{i=1}^{i=n} \sum_{j=1}^{j=n} |y_i - y_j|}{2n^2 \underline{y}}, \tag{1}$$

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along with the usual explanation (following Gini himself) that its value is 1 if one person has all the wealth and is zero if wealth is equally distributed. The students objected that they could not solve the toy example problems with small n that they had been assigned.

After a certain amount of 'redo your calculations' and a lot of head scratching we realised that of course they could not make the problems work because G^L is not 1 when a single person (in a finite population) has all the wealth. For example, where $n = 2$, and one has all the wealth, $G^L = 0.5$.

Our head scratching included a return to his original paper (Gini 1914). There he defined what he called his "concentration ratio" as the sum of the absolute differences among the (unique non-identical) pairs in the population, which we call Δ, or

$$\Delta \equiv \sum_{i=j+1}^{i=n} \sum_{j=1}^{j=n-1} |y_i - y_j| \,, \tag{2}$$

divided by the total number of such pairs, relative to mean wealth, multiplied by one half, or

$$G = \frac{\Delta}{n(n-1)/2} \frac{1}{\underline{y}} \frac{1}{2} = \frac{\Delta}{n(n-1)} \frac{1}{\underline{y}}. \tag{3}$$

From Equation 3 we can see that this is the mean difference among all pairs in the population (the first term in the expression in the middle of Equation 3) divided by the mean value of y, giving us the "relative mean difference", times one half. A feature of this measure is, Gini pointed out, that it satisfies the condition that it varied from one (his "maximum concentration") to zero ("minimum concentration"), as also shown by Angus Deaton (1997).

And Gini showed that in an infinite population this quantity is equal to the area between the (then newly invented) Lorenz curve and the perfect equality line divided by one-half. For an example of a finite $n = 14$ population he provided the appropriate step functions for both the perfect equality "line" and the Lorenz curve. Consistent with our paper, we designate the Lorenz curve-based spatial version of the Gini coefficient as G^L and Gini's version of the concentration ratio in his Equation 11 (from his 1914 paper, which is also our Equation 3 above) as G. As Debraj pointed out, Gini also provided alternative measures (and many more have been proposed since). In their notes, Debraj and Rajiv use G' to refer to our G and G to refer to our G^L.

Maybe the differences that we have aired in this exchange with Debraj and Rajiv stem from our differing perspectives on the relationship between one's ethics and the choice of an appropriate measure of inequality. Debraj writes, "Each measure must be evaluated by the core ethical axioms they satisfy, and we then need to refer to our own ethical system to determine which set of axioms fits the best".

Referring to the Lorenz curve Debraj writes:

> Its foundation is based on a fundamental set of axioms: population neutrality and income neutrality as well as the transfers principle of Pigou and Dalton ... as contributions to a *welfare economics* foundation for inequality measurement, these are the key axioms, and all further explorations stem from them – or should.

We agree that this is one way to choose among competing measures. But we have been motivated, instead, primarily by the desire to measure inequality as it is experienced by the members of a society, hence our term for G: "experienced inequality".

Because one's ethical stance on the distribution of wealth cannot be indifferent to how it is experienced by the members of a population, our measure could be the basis of a normative evaluation, perhaps using empirically measured inequality aversion in the population as the appropriate metric (Fehr and Schmidt 1999). But it might also be an entirely descriptive measure for understanding such things as subjective well-being, stress, alienation, and political attitudes.

Debraj writes, "there are axioms, and then there are axioms". Our view of experienced inequality suggests an addition to the list: **Axiom: Economic inequality is social, that is, it is a relationship between or among people.**

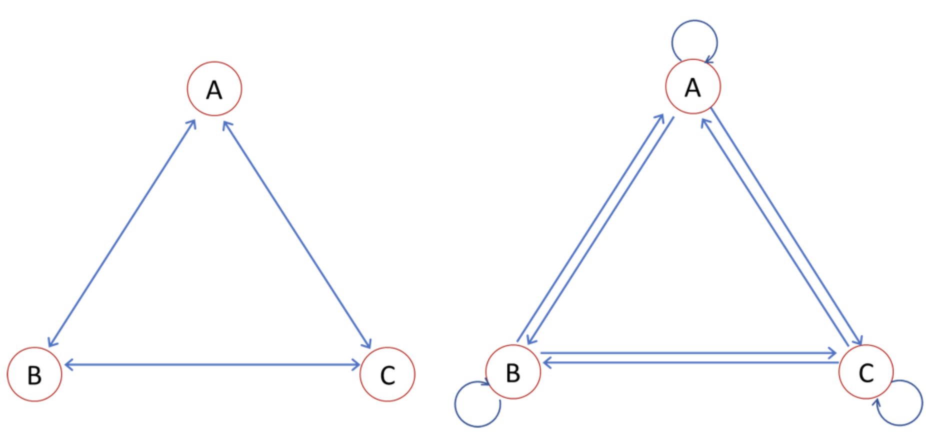
Experienced Inequality

We agree with Debraj and Rajiv that there is no single right way to measure inequality. Which measure one uses depends of course on the question for which the inequality measure is to provide an answer. This often turns on what it is about inequality that one wishes to capture. In Bowles and Carlin (2020), we cite the polarization index that Joan Esteban and Debraj developed as an example of a different measure of inequality developed for a specific purpose, to illuminate social conflict (Esteban and Ray 1994). And we cited the Ross *et al.* (2018) paper on the polygyny threshold, showing that the Gini coefficient (either variant) fails to capture the expected relationship between wealth inequality and polygyny, suggesting the need for alternative measures.

We also illustrated the Gini coefficient, that is, G, as a statistic describing inequality on a complete network the edges of which are the differences in wealth between all pairs of network nodes. Both in teaching and in testing our own intuitions, we found it helpful to let inequality be about the edges of the network rather than the nodes. This means it is about pairwise differences in wealth, that is about relationships, not how much wealth each individual has. An example comparing the two approaches is in Figure 1, with the inequalities counted in Equation 3 on the left and in Equation 1 on the right.

We made the case that, represented in this way, G captures important aspects of the way inequality is experienced by the members of a society in which people are aware of the wealth levels of everyone else. We also find the approach insightful as it allows us to measure experienced inequality in a society in which the relevant comparison set of individuals comprises not all others, but instead all others to whom one is connected in the network. For example, in a hub and spokes network with a wealthy individual at the center and the peripheral nodes connected only to the center, there is much more experienced inequality than in a complete network for any given set of nodes and levels of wealth at each node.

Figure 1
Experienced differences (left panel) and the edges used in the conventional Lorenz curve-based measure (right panel)



If Nodes A, B, and C in Figure 1 have wealth 10, 4, and 3, the Gini coefficient, given by Equation 3 using the network representation in the left panel, is 0.412. Using the network representation on the right (that is, Equation 1), however, the Gini is estimated as 0.275, which needs to be multiplied by $n/(n-1)$ or 1.5 to get the Gini coefficient restricted to the differences among actual pairs in the population (excluding the three "self-on-self" zero differences, the circular arrows in our figure), as pointed out by Yitzhaki and Schechtman (2013).

To sharpen our intuitions about inequality seen as pairwise differences among members of a population, suppose that every day, individuals are randomly paired to interact – economically, socially, in religious observance, and so on – with another member of the society. In the complete network representation, one of each individual's edges is selected at random. We are interested in the frequency over a great many such random pairings with which a member of the population interacts with a person of similar or different wealth, as we believe that the nature of these interactions will differ in important ways if the wealth differences are significant.

To explore looking at inequality in this way, let's consider an economy with just two wealth levels, which, without loss of generality, we will set to zero and to some positive number, which is total wealth y divided equally among r rich members of the total population of n. Total wealth is some given level of mean wealth multiplied by the size of the economy or $y \equiv \underline{y}n$. Then, the only unequal pairs in the population are the r wealth holders interacting with the $n - r$ wealthless individuals with both members of the pair experiencing a wealth difference of y/r. So, we have $\Delta = r(n-r)\underline{y}n/r$ and Equation 3 becomes

$$G = \frac{\Delta}{n(n-1)}\frac{1}{\underline{y}} = \frac{r(n-r)\underline{y}n/r}{n(n-1)}\frac{1}{\underline{y}} = \frac{n-r}{n-1}. \tag{4}$$

From Equation 4 we confirm that irrespective of the population size, $G = 1$ when one person has all the wealth (that is, $r = 1$) and that inequality declines as wealth is shared among a larger number of rich, that is, increasing r.

We can also see that holding constant the number of the wealthy, r,

$$\frac{dG}{dn} = \frac{1-G}{n-1} > 0 \text{ for } G < 1. \tag{5}$$

So G increases as the propertyless class increases (holding constant the size of the wealthy class). In the limiting case where a single person owns all of the wealth, increasing the number of propertyless individuals does not affect the G. It is this limiting case, where $r = 1$ so $G = 1$, that Debraj labels as the "absurd consequence" of measuring inequality by G.

Finally, to focus on relationships – the edges of the network rather than the nodes – we calculate the fraction δ of all pairs in which the two have different wealth levels,

$$\delta = \frac{r(n-r)}{n(n-1)/2} = \frac{2r}{n}G, \tag{6}$$

from which we see that if the population is divided into classes of equal size (so that $2r/n = 1$), then G *is* the fraction of the interactions in which the members of the pair have different wealth.

Note also that if there is just one wealth owner and one wealthless person, all of the interactions are unequal and $\delta = 2/n = 1$ (there is just a single interaction). But as the propertyless class increases in size the fraction of all interactions that are unequal falls.

How does it come about, then, that G increases with n (for a fixed number of rich households)? The answer is that, as n rises (with r fixed), total wealth increases proportionally (recall that $y \equiv \underline{y}n$), so the mean wealth of the rich increases, and as a result the wealth difference between members of an unequal pair (that is, $\underline{y}n/r$) also increases.

Is invariance to population replication a desideratum for a measure of inequality?

An attractive feature of the now-common variant of the Gini coefficient (Equation 1, G^L) for populations of any size is that it is equal to the area between the perfect equality line and the Lorenz curve, divided by one-half. But, as Equation 1 and the right panel of Figure 1 make clear, G^L includes the "equality" of one's own wealth with one's own wealth (what we term the self-on-self "fictive zeros") in the measure of societal inequality, violating our "economic inequality is social" axiom. As the term is routinely used, "social inequality" concerns relationships between people; in a one-person society (where $G^L = 0$) the "level of inequality" is meaningless, not zero. We find this aspect of G^L to be a reason not to use it where it differs appreciably from G.

The payoff to the inclusion of the self-on-self comparisons in G^L is said to be that, defined in this way, the measure conforms to another axiom ("population symmetry", in Rajiv's contribution), namely, that the measure of inequality should be invariant to replication of its members, so that inequality would be the same (maximal concentration) in a two-person society in which one held all of the wealth and in a four-person society in which two people equally shared all of the wealth or one thousand equally shared the wealth with another one thousand wealthless, and so on.

From the standpoint advanced by Debraj and Rajiv, it is worth paying the price of including the counterintuitive fictive zeros to satisfy "invariance to population replication", an axiom that is thought to be sufficiently intuitively appealing to justify setting aside the problem of including these self-on-self comparisons in a measure of societal inequality.

However, from the perspective of experienced inequality, invariance to population replication is a bug not a feature of Equation 1. Let's think about three economies with two, four, and six people, in each of which, half of the population owns all of the wealth in equal shares and as before, total wealth is proportional to population size. Is it obvious that the three economies are "equally unequal" such that the level of equality is unaffected by population replication? Does the fact that our measure represents the level of inequality as greater in the two-person society than in the four- and six-person societies, in violation of the population replication axiom, disqualify it as an inequality measure? Let's see what is involved.

In all three societies, G is also equal to the fraction of all interactions that are between people of differing wealth levels (because $2r = n$ from Equation 5, $\delta = G$). This falls from 100 *per cent*, when $n = 2$ – the only interactions are between rich and poor – to $2/3$ for $n = 4$, and to $3/5$, when $n = 6$. The wealth difference between the members of the unequal pairs is unchanged as the population replicates (because n/r by design is unchanging). The only thing that has changed is that in the larger population people experience interactions with others of the same wealth level a substantial fraction of the time. Our intuition is that G correctly shows that experienced inequality in larger societies is less, and that as a result the invariance to the population replication axiom is unappealing or at least not so intuitive that its violation would disqualify G.

To understand how counterintuitive the invariance to population replication is, we now imagine a contrived process of population replication that would not affect experienced inequality, G, thus satisfying the axiom. Return to the above example, but as the number of rich and poor increases from two each to four each, suppose the two poor and the two rich individuals form two-person households, one rich, the other poor, and similarly when the population grows to $n = 6$, there are just two three-person households, one rich and the other poor. Then because of the fictive formation of these two ever-larger households, between-household wealth differences relative to average household wealth would remain unchanged and so, as a result, would the Gini coefficient (G) if measured on this contrived example of population replication, satisfying the invariance to population replication axiom.

The reason is that the additional within-class pairings of equals – which leads G to fall as both n and r rise proportionally in the more realistic representation of population expansion – would all be within these fictive households and hence would be ignored. But why should these within-class equal pairings be ignored in the more plausible case that population growth occurs through the multiplication of the number of households, not the size of households? We cannot think of a reason.

Let's now reconsider what Debraj finds "even more absurd" about G, namely, that a thousand-person society with just two wealth holders is more equal than a two-person society with one person holding all the wealth. In the former, the vast majority of interactions are among people with the same wealth ($1-\delta = 1-(2\times 998)/(1000\times 999\times 1/2) = 0.996$), while in the latter, none is. This does not settle which is "really" more unequal. This is because where they exist, the wealth differences in the thousand-person society are much greater than in the two-person society (500 times mean wealth rather than twice). But the example does suggest that it is far from absurd to think that the thousand-person society with two owning all the wealth would be experienced (or even perceived by an outsider) as in some sense more equal than the two-person society in which one person has all the wealth.

We are not saying that Debraj's intuition is incorrect (how could it be?). But it is far from obvious to us. And competing and quite different intuitions might also be appealing.

The above discussion also recommends a network representation of inequality if capturing the experience of inequality is an objective. We have motivated our examples by a random pairing environment equivalent to a complete network. But societies differ greatly in who interacts with (or even is aware of) whom. For example, if the social structure in question is a hub and spokes network with the wealth holder at the center, then the fraction of one's interactions that are with someone of a different wealth level (all of them, $\delta = 1$) is a constant as the size of the network grows, and as a result, δ will be greater (for $n > 2$) than in the complete network that we have used above as our illustration.

Notes

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[2] https://www.core-econ.org/

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